EFFECT OF THREAD NUMBER ON THE PUMPING CAPACITY OF A HELICAL SCREW AGITATOR

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Theoretical regression functions of modifying factors in dependence on geometrical simplexes of a helical agitator are presented, based on an analysis of liquid flow through the draught tube with a rotating helical agitator. A quantitative relation for the liquid flow rate through the draught tube calculated from the mathematical model of creeping flow through a infinitely long draught tube is modified by their use. This is done for the draught tube of finite length with the helical agitator having a finite and small number of threads.

Draught tube with a rotating helical agitator (Fig. 1) is the principal part of this type of mixing unit. In this part of the mixing unit the helical agitator transfers momentum to the liquid. Liquid is agitated in a complex manner and, if walls of the draught tube serve as heat transfer areas, heat transfer takes place as well.

Theoretical analysis of liquid flow or heat transfer in the draught tube is partly made difficult by the complicated geometrical shape of the helical agitator. Helical agitator is formed by a shaft with a wound agitator blade which is part of the helical surface area. Geometrical parameters of this part of helical surface area – slope of helical area s, radii r_1 and r_2 which are limiting the area in the radial direction in respect to the agitator, number of threads of the blade z and its thickness e – are of course affecting the mentioned transport processes in the draught tube. It is therefore necessary that the mentioned geometrical parameters of the helical area be implicitely included in the mathematical model.

In the preceding papers[1,2], such mathematical model with implicitely included mathematical parameters s, r_1, r_2 and e of the helical agitator for creeping flow of Newtonian liquid was presented and solved. But there it was assumed that the length of the draught tube did not affect the liquid flow. Thus the actual effect of the length of the draught tube *i.e.* the effect of geometrical parameter z representing the number of threads of the helical agitator cannot be determined on basis of that mathematical model.

As concerns the characterization of the whole mixing system, the knowledge of the pumping capacity of the helical agitator is very important. By its use *e.g.* the mixing efficiency may be characterized[3], and the optimal geometrical arrangement can be determined of the whole mixing system from the hydrodynamic point of view[4] as well as from the view of heat transfer[5] between the wall of the mixing unit and the mixed liquid.

Since the pumping capacity of the helical agitator is an important parameter and because the effect of geometrical parameters of the helical agitator s, r_1, r_2 and e on pumping capacity of the agitator may be determined by numerical solution of the mathematical model of liquid flow in an infinitely long draught tube with a rotating helical agitator, the effect of number of threads of the helical agitator on its pumping capacity can be advantageously determined by a relatively simpler method — by modification of results calculated from the mathematical model by use of empirical regression functions which can be evaluated from suitable experimental data.

Procedure resulting in determination of the effect of number of threads of the helical agitator on its pumping capacity is presented in this paper. The proposed theoretical regression functions are analogous with those used in the paper[6] where the effect of the length of two planes of finite length (modelling a duct in an extruder) on flow rate between the planes is determined by an exact method.

THEORETICAL

Modelling of the liquid flow through the draught tube with a rotating helical agitator is possible (under the assumption that the clearance between the edge of the mixer blade and the draught tube is small) by liquid flow through a helical rectangular duct with a movable wall[1]. From solution of the mathematical model of creeping flow of Newtonian liquid through this rectangular duct, a linear dependence of the dimensionless liquid flow rate Q_{KR} on dimensionless pressure gradient P_{KR} has been obtained[2]

$$Q_{KR} = A + BP_{KR}\,, \tag{1}$$

where A and B are dimensionless parameters which in general are functions of dimensionless geometrical simplexes ξ_i

$$A = A(\xi_i)\,, \quad B = B(\xi_i)\,, \tag{2}$$

where ξ_i for $i = 1, 2, 3$ is $\xi_1 = s/h$, $\xi_2 = r_1/h$, $\xi_3 = e/h$ and $h = r_2 - r_1$.

For finite number of threads z of the helical agitator it is necessary to modify relation (*1*) into the form

$$Q_{KR} = F_A A + F_B BP_{KR}\,, \tag{3}$$

where F_A, F_B are dimensionless modifying factors which are in general dependent on the geometrical simplexes ξ_i and z

$$F_A = F_A(\xi_i, z)\,, \quad F_B = F_B(\xi_i, z)\,, \quad i = 1, 2, 3\,. \tag{4}$$

In general, functional relations (*4*) of modifying factors F_A and F_B on geometrical simplexes ξ_i and z of the helical agitator can be expressed in a form of theoretical regression functions. These theoretical regression functions may be for creeping flow in the draught tube obtained by the following procedure.

For pressure drop in the end sections (*i.e.* in the entry and exit sections where the dynamic equilibrium between the pressure and viscous forces does not exist and so the velocity profile is not developed) of the helical rectangular duct which is modelling the liquid flow through the draught tube with a rotating helical agitator obviously holds (see Fig. 2)

$$P_1 - P_1' + P_2' - P_2 = 2\pi z_2\, \mathfrak{P}(1/F_B')\,, \tag{5}$$

where $\mathfrak{P}$ is the components of pressure gradient in the duct section with a developed velocity profile and F_B' is the factor modifying the value of the pressure gradient $\mathfrak{P}$ for the end sections.

On the basis of relation (*5*) for the over-all pressure drop at liquid flow through the helical rectangular duct with a movable wall we may write

$$P_1 - P_2 = 2\pi z_1\, \mathfrak{P} + 2\pi z_2\, \mathfrak{P}(1/F_B') \tag{6}$$

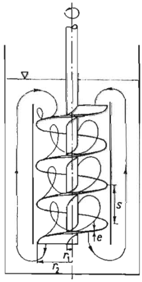

FIG. 1

Mixing Unit with Helical Screw Agitator Rotating in Draught Tube

Basical geometrical parameters of the helical screw agitator and typical trajectories of the agitated liquid are indicated.

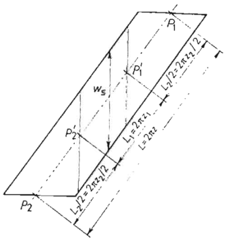

FIG. 2

Sectional View of the Helical Rectangular Duct Formed by the Helical Screw Agitator and Draught Tube in Plane r = const and

$$P_1 - P_2 = 2\pi z\, \mathfrak{P}(1/F_B)\,. \tag{7}$$

For the lengths as indicated in Fig. 2 we have

$$2\pi z_1 = 2\pi z - 2\pi z_2\,. \tag{8}$$

On combining relations (6)–(8) the relation can be obtained

$$(z/F_B) - z = (z_2/F'_B) - z_2\,. \tag{9}$$

The right hand side of Eq. (9) is obviously constant for a given geometrical arrangement of the helical rectangular duct (z_2 is obviously the length of end sections which is together with the modifying factor F'_B independent on the over-all length of the helical rectangular duct). In general the relation (9) may be written

$$(z/F_B) - z = f(\xi_i)\,, \quad i = 1, 2, 3 \tag{10}$$

from which the following relation is obtained

$$F_B = [1 + f(\xi_i)\, z^{-1}]^{-1}\,, \quad i = 1, 2, 3\,. \tag{11}$$

The functional dependence of $f(\xi_i)$ from relation (11) can be only chosen. With regard to the obtained experimental results (Table I) this dependence is used in the form of a product of exponential geometrical simplexes ξ_i. The theoretical regression function for the modifying factor F_B thus has the form

$$F_B = [1 + C(\prod_{i=1}^{3} \xi_i^{a_i})\, z^{-1}]^{-1}\,, \tag{12}$$

where C, a_i – $i = 1, 2, 3$ are constants.

Theoretical regression functions for the modifying factors F_A can be obtained in a similar way as the above obtained theoretical regression function for the modifying factor F_B.

It is obvious from the linear relations (1) and (3) that for a certain value of the dimensionless pressure number P_{KR} values of flow rates Q_{KR} calculated from these relations will be the same. Thus it holds

$$A + B(P_{KR})_{eq} = F_A A + F_B B(P_{KR})_{eq}\,, \tag{13}$$

where $(P_{KR})_{eq}$ is the value of P_{KR} for which the flow rates Q_{KR} calculated from relations (*1*) and (*3*) are equal.
Further it may be written

$$K = B(P_{KR})_{eq}/A\,. \tag{14}$$

On combining relations (*13*) and (*14*) it is possible to write

$$F_A = 1 + K(F_B - 1)\,. \tag{15}$$

According to relation (*14*) the variable K is in general a function of geometrical simplexes ξ_i for a certain value of dimensionless pressure number $(P_{KR})_{eq}$. Relation (*15*) has in general the form

$$F_A = 1 + f(\xi_i)\,[F_B - 1]\,, \quad i = 1, 2, 3\,. \tag{16}$$

The general functional modifying factor F_A in relation (*16*) is expressed similarly as for the theoretical regression function for modifying factor F_B with regard to the obtained experimental results (see Table I) in the form of a product of exponential geometrical simplexes ξ_i. The theoretical regression function for the modifying factor F_A has thus the form

$$F_A = 1 + D\left(\prod_{i=1}^{3} \xi_i^{b_i}\right)[F_B - 1]\,, \tag{17}$$

where D, b_i – $i = 1, 2, 3$ are constants. Validity of relations (*12*) and (*17*) is limited only to creeping flows of Newtonian liquids *i.e.* approximately for $Re < 20$ (see[7]) and for such geometrical arrangements for which the radius of the blade of the helical agitator and the internal radius of the draught tube differ only very slightly (less than by 3%).

RESULTS

Values of constants C, D, a_1 and b_1 for the theoretical regression functions (*12*) and (*17*) of modifying factors can be evaluated by the method of least squares from the data given in Table I. These results are statistically evaluated from the experimental liquid flow rates through the draught tube with a rotating helical agitator in dependence on the pressure drop of the mixing vessel (pressure drop is in a dimensionless form expressed by the pressure number P_{KR}) and on the varying geometrical simplex of the helical agitator ξ_1. Values of the geometrical simplex ξ_2 were constant during the experiments. The effect of the geometrical simplex ξ_3 is not evaluated as it is insignificant in the case of experimental as well as in the majority of practical geometrical arrangements of the helical agitator.

The theoretical regression functions (*12*) and (*17*) are not linear in respect to evaluated constants. By introducing new variables p and q by relations

$$p = \lg\left[\frac{z(F_B - 1)}{F_B}\right], \tag{18}$$

$$q = \lg\left[\frac{(1 - F_A)}{(F_B - 1)}\right], \tag{19}$$

the theoretical regression functions (*12*) and (*17*) may be, in respect to the evaluated constants, linearized. The empirical regression functions for modifying factors F_A and F_B can be obtained by evaluating the constants C, D, a_1 and b_1 by the method of least squares from the experimental data given in Table I. The following relations then hold

$$F_A = 1 - 0{\cdot}0063\xi_1^{2.25}[F_B - 1], \tag{20}$$

$$F_B = [1 - 0{\cdot}55\xi_1^{-0.63}z^{-1}]^{-1}. \tag{21}$$

TABLE I

Experimental Values F_A and F_B for Evaluation of Empirical Regression Functions (*20*) and (*21*)

Duct simplexes	A^a	B^a	z	$(F_AA)^b$	$(F_BB)^b$	$F_A = \frac{(F_AA)}{A}$	$F_B = \frac{(F_BB)}{B}$	$F_A{}^c$	$F_B{}^c$
$\xi_1 =$ 1·43			3	0·38	−0·08	1·013	1·290	0·999	1·185
$\xi_2 =$ 0·3	0·375	−0·062	—	—	—	—	—	—	—
$\xi_3 =$ 0·15			6·2	0·38	−0·07	1·013	1·129	0·999	1·082
$\xi_1 =$ 3·0			1	0·78	−0·23	0·906	1·483	0·971	1·378
$\xi_2 =$ 0·3	0·865	−0·155	3	0·88	−0·18	1·017	1·161	0·997	1·100
$\xi_3 =$ 0·18			6·1	0·88	−0·16	1·017	1·032	0·999	1·047
$\xi_1 =$ 4·39			1·1	0·85	−0·20	0·830	1·075	0·961	1·244
$\xi_2 =$ 0·3	1·024	−0·193	3·1	1·00	−0·21	0·976	1·129	0·995	1·074
$\xi_3 =$ 0·16			6·1	1·03	−0·21	1·005	1·129	0·998	1·036
$\xi_1 =$ 8·25			1	0·67	−0·20	0·814	1·075	0·889	1·160
$\xi_2 =$ 0·3	0·823	−0·186	—	—	—	—	—	—	—
$\xi_3 =$ 0·23			3	0·77	−0·21	0·935	1·129	0·988	1·049
$\xi_1 =$ 14·1			—	—	—	—	—	—	—
$\xi_2 =$ 0·3	0·511	−0·137	1	0·42	−0·17	0·821	1·240	0·749	1·109
$\xi_3 =$ 0·26			—	—	—	—	—	—	—

[a] Values calculated from the mathematical model of liquid flow through rectangular duct[2]. [b] Experimental data[7]. [c] Values calculated from Eqs (*20*) and (*21*).

LIST OF SYMBOLS

A, B	parameters in Eq. (*1*)
a_i, b_i	constants
e	thickness of the helical agitator blade
F_A, F_B	modifying factors in Eq. (*3*)
F'_B	modifying factors in Eq. (*6*)
$h = r_2 - r_1$	duct height
K	parameter defined by Eq. (*14*)
L	duct length
L_1	duct length with fully developed velocity profile
L_2	length of end sections of the duct
$P_{KR} = p_s h/2\pi z \mu u$	pressure number
$(P_{KR})_{eq}$	value of pressure number for identical flow rates calculated from Eqs (*1*) and (*3*)
P_1	inlet pressure in the duct axis
P_2	outlet pressure in the duct axis
P'_1	pressure in the duct axis at the beginning of section with fully developed velocity profile
P'_2	pressure in the duct axis at the end of section with fully developed velocity profile
$p_s = (P_1 - P_2)/L$	pressure drop in duct
q	flow rate in duct
$Q_{KR} = q/h^2 u$	flow rate number
r_1	radius of shaft of the helical agitator
r_2	radius of blade edge of the helical agitator
$\mathrm{Re} = hu\varrho/\mu$	Reynolds number
s	pitch of blades of the helical agitator
u	relative velocity of duct wall in axial direction
z	number of threads of helical agitator
z_1	number of threads of helical agitator corresponding to the length of duct with fully developed flow
z_2	number of threads of helical agitator corresponding to the length of duct with fully developed flow
ξ_i	geometrical simplexes
μ	dynamic viscosity
$\mathfrak{P}$	pressure gradient in section of duct with fully developed velocity profile
ϱ	density

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